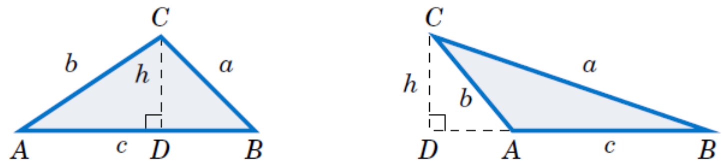
헤론의 공식

$A = \sqrt{s(s-a)(s-b)(s-c)}$, $\left(s = \frac{a+b+c}{2} \right)$

증명



사용공식

$\overline{AD} = b\cos A$ $\overline{AD} = b\cos(180 - A) = -b\cos A$ ----- ①

$\overline{AD}^2 = b^2(\cos A)^2$

$a^2 = b^2 + c^2 - 2bc\cos A \rightarrow \cos A = \frac{b^2+c^2-a^2}{2bc}$ ------ ②

$a^2 - b^2 = (a+b)(a-b)$ ------ ③

$s = \frac{a+b+c}{2}$ 라 하자

$A = \frac{1}{2}ch$

$A^2 = \frac{1}{4}c^2h^2$

$h^2 = b^2 - (\overline{AD})^2 = b^2 - (b\cos A)^2$

$h^2 = b^2 - b^2(\cos A)^2 = b^2(1-(\cos A)^2) = b^2(1+\cos A)(1-\cos A)$

$A^2 = \frac{1}{4}c^2b^2(1+\cos A)(1-\cos A)$

$1+\cos A = 1 + \frac{b^2+c^2-a^2}{2bc} = \frac{2bc+b^2+c^2-a^2}{2bc} = \frac{(b+c)^2-a^2}{2bc} = \frac{((b+c)+a)((b+c)-a)}{2bc}$

$= \frac{(a+b+c)(b+c-a)}{2bc}$,

$1-\cos A = 1 - \frac{b^2+c^2-a^2}{2bc} = \frac{2bc-b^2-c^2+a^2}{2bc} = \frac{a^2-(b-c)^2}{2bc} = \frac{(a-(b-c))(a+(b-c))}{2bc}$

$= \frac{(a-b+c)(a+b-c)}{2bc}$.

$A^2 = \frac{1}{4}c^2b^2 \cdot \frac{(a+b+c)(b+c-a)}{2} \cdot \frac{(a-b+c)(a+b-c)}{2}$

$A^2 = \frac{(a+b+c)}{2} \cdot \frac{(b+c-a)}{2} \cdot \frac{(a-b+c)}{2} \cdot \frac{(a+b-c)}{2}$

$$A^2 = \frac{a+b+c}{2} \cdot \frac{a+b+c-2a}{2} \cdot \frac{a+b+c-2b}{2} \cdot \frac{a+b+c-2c}{2}$$

$$A^2 = s(s-a)(s-b)(s-c)$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$